MATHEMATICS AND TRANSCENDENTALISM

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The whole matter of geometry is transcendental. —*Theodore Parker*

1. Introduction

Transcendentalism is based on the idea of Reason, the word Samuel Taylor Coleridge used to describe a faculty by which humans can discern truth without regard to the external world. By looking within, the Transcendentalists claimed, we can discover the nature of God. But why should we believe that this is possible? Why should we believe that there *is* such a faculty of Reason? In this essay, I argue that Immanuel Kant, Coleridge, and the Transcendentalists used mathematics as an example to prove the existence of Reason (known in more technical terms as "synthetic *a priori* reasoning"). Furthermore, I argue that the reason why mathematics was chosen to exemplify Reason is that mathematics is particularly effective at describing the external world; the facts that one can deduce using mathematics hold true (as far as we can determine) in the external world, and two different mathematicians have never proven contradictory results.[1] But the method that mathematicians use to ensure that their reasoning is valid — namely, by requiring that their arguments be formalized into structured proofs that must convince other, skeptical, mathematicians —

1. Essentially all working mathematicians would agree that contradictory results have never been proven. However, in view of criticisms such as those by Lakatos (Imre Lakatos, *Proofs and refutations*) I should be a little more precise. It is not uncommon for mathematicians to *think* they have proven a statement that contradicts a previously proven result, but in every instance further examination shows that the 'proof' of one of the two statements contains an error — or, more interestingly, the conflicting arguments reveal that a word or concept has not been properly defined or thoroughly understood. Once the concepts have been examined, the apparent contradictions have always been resolved.

was *not* embraced by the Transcendentalists. Likewise, the Transcendentalists were not overly concerned with the basic method that scientists use to test their ideas: conducting repeatable experiments subject to empirical validation. (Indeed, Philip Gura describes Coleridge's Reason as "an internal principle not subject to empirical proof."[2]) Thus, using mathematics as an exemplar of Reason is a misdirection, because the very aspect of mathematics that ensures that its results are true is the aspect that the Transcendentalists dismissed. This part of my argument is found in Sections 2 and 3.

In Section 4, I explain how a result of Euclidean geometry used by Kant, Coleridge, Theodore Parker, and others as an example of an indisputable fact arrived at by Reason is, in fact, not true in the non-Euclidean geometries discovered in the 19th century. For nearly two millennia, mathematicians had been unable to prove this geometric result without assuming Euclid's fifth postulate — the "parallel postulate." With the discovery of non-Euclidean geometry, it became clear that the intuition that this result must be true in all forms of geometry was in fact incorrect. Thus, the very example used by the Transcendentalists as a truth arrived at by Reason turned out to be the key player in a mathematical conflict between intuition and rigorous argument, an example that demonstrates the necessity of viewing proofs as dialogues within a community.

A frequent criticism of the Transcendentalists — sometimes made also against present-day Unitarian Universalists — is that they focussed so much on self-culture and on internal spiritual development that they dismissed or ignored the communal aspects of spirituality. My arguments show that this focus on the internal, to the exclusion of the external and the communal, appears in — and weakens — the arguments

2. Philip F. Gura, *American transcendentalism: A history* (New York: Hill and Wang, 2007), 53.

that provide the very foundation of the philosophical justification for Transcendentalism. In Section 5 I close by discussing how present-day Unitarian Universalists might avoid this philosophical weak spot of the Transcendentalists.

2. Mathematics as evidence for Reason

As Gura explains,[3] the early Transcendentalists were inspired by the ideas about intuitive perception of truth set forth by Kant and his followers, especially Coleridge. Coleridge's idea of Reason — a faculty by which a person "is capable of quickening inter-communication with the Divine Spirit"[4] — fit well with the thoughts that William Ellery Channing expressed in his 1828 sermon *Humanity's likeness to God*:

> That man has a kindred nature to God, and may bear most important and ennobling relations to him, seems to me to be established by a striking proof. This proof you will understand, by considering, for a moment, how we obtain our ideas of God. Whence come the conceptions which we include under that august name? Whence do we derive our knowledge of the attributes and perfections which constitute the Supreme Being? I answer, we derive them from our own souls.[5]

In order to put arguments of this kind on a sound footing, and to establish some justification for the existence of this faculty of Reason, philosophers like Kant and Coleridge had to provide some evidence for their claim that there is a way of obtaining new knowledge that does not depend on the senses. The most convincing argument that they could make to this end was to point to an existing, universally respected area of human thought that obtains verifiable truths without recourse to the senses. For this purpose, Kant and Coleridge turned to mathematics.

3. Ibid., Chapter 2.

4. Samuel Taylor Coleridge, *Aids to reflection, in the formation of a manly character, on the several grounds of prudence, morality, and religion*, First American edition (Burlington: C. Goodrich, 1829), URL: http://books.google.com/books?id=gZUKAAAAYAAJ, 137.

5. Quoted in Gura, see n. 2, 15.

Of course, some argument must be made to show that mathematics is in fact not based on the senses. As Daniel J. Cohen points out,[6] Coleridge devotes three chapters of his *Logic* to a discussion of the epistemology of mathematics, partly in order to refute philosophers like David Hume, who claimed that mathematics is based on analysis of our observations of the world: that we know that $2 + 3 = 5$ because we have taken two apples, and added three more to the pile, and found that we then have five apples. But, Coleridge asks,[7] what if we consider a larger sum, like 35,942,768,412 + 57,843,647? Surely we have never seen such a sum represented in physical objects and counted out the result. From arguments like these, Coleridge concludes that mathematics is synthetic *a priori* reasoning. Here *synthetic* means that it creates new knowledge from old, as opposed to *analytic* reasoning, which involves dissecting previously known things — in Coleridge's memorable phrase, "By the analytic process we know what we know *better*; by the synthetic we know more."[8] Likewise, here Coleridge uses *a priori* to mean "*a priori* to the evidence of the senses."

Coleridge is paraphrasing the introduction to Kant's *Critique of Pure Reason* throughout this discussion. Kant begins his introduction with the summary: "Mathematical judgements are always synthetical [...] [P]roper mathematical propositions are always judgments *a priori*."[9]

Many pages later, Kant writes:

> The science of Mathematics presents the most brilliant example of the extension of the sphere of pure reason without the aid of experience.

6. Daniel J. Cohen, *Equations from God: Pure mathematics and Victorian faith*, Johns Hopkins Studies in the History of Mathematics (Baltimore: The Johns Hopkins University Press, 2007), 34–35.

7. Samuel Taylor Coleridge, *Logic*, ed. J. R. de J. Jackson, vol. 13, The Collected Works of Samuel Taylor Coleridge (Princeton: Princeton University Press, 1981), 201–202.

8. Ibid., 174.

9. Immanuel Kant, *Critique of Pure Reason*, Translated by J. M. D. Meiklejohn (New York: P. F. Collier and Son, 1901), URL: http://books.google.com/books?id=7bRychF0y0EC, 52–53.

> Examples are always contagious; and they exert an especial influence on the same faculty, which naturally flatters itself that it will have the same good fortune in other cases as fell to its lot in one fortunate instance. Hence pure reason hopes to be able to extend its empire in the transcendental sphere with equal success and security, especially when it applies the same method which was attended with such brilliant results in the science of Mathematics. It is, therefore, of the highest importance for us to know, whether the method of arriving at demonstrative certainty, which is termed *mathematical*, be identical with that by which we endeavor to attain the same degree of certainty in philosophy, and which is termed in that science *dogmatical*.[10]

Kant here explicitly points to the success and "brilliant results" of mathematics. It is important not only that mathematics be synthetic *a priori* reasoning, but also that it be correct. If mathematics is correct, we will know that our synthetic *a priori* reasoning in philosophy may also be correct.

The fact that mathematics *works* — that investigations and computations carried out entirely in the mind are reflected so well by the physical world — has puzzled and inspired philosophers since Plato.[11] Cohen[12] gives a good exposition of how Victorian scientists, theologians, and the public in general were captivated by a spectacular demonstration of the effectiveness of mathematics in 1846. In the 1840's, astronomers deduced the existence of the planet Neptune from aberrations in the orbit of Uranus. From careful measurements of these aberrations, and from extensive mathematical computations, two different mathematicians — John Couch Adams in England and Urbain Le Verrier in France — predicted where in the sky Neptune should be visible. On September 23, 1846, an astronomer in Berlin looked at the region of the sky predicted by Le Verrier and Adams, and found Neptune.

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10. Ibid., 255.

11. Articles are still being written about the "unreasonable effectiveness" of mathematics; see, for example, the much-cited article of Wigner (Eugene P. Wigner, "The unreasonable effectiveness of mathematics in the natural sciences") and the many papers in the mathematical literature published after Wigner that have the words "unreasonable effectiveness" in their titles.

12. Cohen, see n. 6, Introduction.

This highly-visible success for mathematics was celebrated in many quarters, and most certainly by scientists. Cohen quotes John Herschel, the president of the Royal Astronomical Society:

> That a truth so remarkable should have been arrived at by methods so different by two geometers, each proceeding in utter ignorance of what the other was doing, is the clearest and most triumphant proof [of how well Newton's laws were represented in the their equations.][13]

(Note how Herschel specifically points out that although the two mathematicians were inspired to use *different* methods, they obtained the same result; their two subjective truths were objectively in accord.) The Scottish optics researcher David Brewster, stressing the independence of mathematical thought from the senses, waxed poetic:

> If man ever sees otherwise than by the eye, it is when the clairvoyance of reason, piercing through screens of epidermis and walls of bone, grasps, amid the abstractions of number and quantity, those sublime realities which have eluded the keenest touch, and evaded the sharpest eye.[14]

Theologians were also impressed, and saw this success as being a consequence of a relation to the Divine. Cohen[15] notes that Cyrus Augustus Bartol, the Unitarian minister of Boston's West Church, a participant[16] in the first meeting of "Hedge's Club," and a major Transcendentalist after the Civil War, wrote:

> [The mathematician] scans these perturbed inclinations more exactly, measures their amount, ascends to their adequate cause, and though that cause still lay darkly ranging on, with to earthly vision undiscernable lustre, he yet predicts its place, and course, and time of arrival into the focus of human sight. His prediction is recorded, to be entertained by some, or incredulously smiled at by others.

13. Quoted in Cohen, see n. 6, 2.
14. Quoted in ibid., 3.
15. Ibid., 5.
16. Gura, see n. 2, 70.

> But lo! in due time the stranger comes as announced, to fulfil this "sure prophetic word" of the divinely inspired understanding of man.[17]

This association of mathematics with the Divine was part of a larger international cultural context. The French philosopher Edgar Quinet, in an 1844 lecture on science and the Catholic Church, wrote of the solutions to mathematical problems:

> Ces pures et incorruptibles formules, qui étaient avant que le monde fût, qui seront après lui, qui dominent tous les temps, tous les espaces, qui sont, pour ainsi dire, une partie intégrante de Dieu, ces formules sacrées qui survivront à la ruine de tous les univers, mettent le mathématicien qui mérite ce nom, en communion profonde avec la pensée divine. Dans ces vérités immuables, il savoure le plus pur de la création; il prie dans sa langue. Il dit au monde, comme cet ancien : « Faisons silence, nous entendrons le murmure des dieux! »[18]
>
> [These pure and everlasting formulas, that *were* before the world was made, and *will be* after it has gone; that govern through all time, and throughout all space; that are, as it were, an essential part of God — these sacred formulas, which will survive the ruin of the universe, put any mathematician worthy of the name in profound communion with divine thought. In these immutable truths he tastes the perfection of creation, and he prays in its language. He says to the world, as was said of old: "Be silent! — that we may hear the whispers of the gods!"]

Quinet's exuberant quotation traveled all the way to America. In 1847 it appeared in a review of an algebra textbook in the Methodist Quarterly Review[19] (!), sandwiched between an article on Greek lexicography and the annual report of the missionary

17. Cyrus Augustus Bartol, "The new planet: Or, an analogy between the perturbations of matter and spirit," *The Monthly Religious Magazine* 4 (1847), URL: http://books.google.com/books?id=Xn8oAAAAYAAJ: 77.

18. Edgar Quinet, "L'ultramontanisme, ou, l'èglise romaine et le société moderne," in *Œuvres complètes de Edgar Quinet* (Paris: Pagnerre, Libraire-Editeur, 1857), URL: http://books.google.com/books?id=rs4TAAAAQAAJ, 186–187.

19. "A treatise on algebra [review]," *The Methodist Quarterly Review* 29 (Apr. 1847): 257–269, URL: https://books.google.com/books?id=xA5BAQAAMAAJ&pg=PA257.

society, and in 1848 the Congregationalist minister Horace Bushnell quoted these words of Quinet in an address to the Phi Beta Kappa society.[20,21]

Of course, none of these florid tributes would have been written had Adams and Le Verrier or their mathematical predecessors predicted incorrectly. The fact that their calculations — and the calculations of untold mathematicians before them — reflected reality was the basis for the association of mathematics with the Divine.

We see that mathematics played an important strategic role for the Transcendentalists, in part due to the respect in which it was held in religious and philosophical circles. The example of mathematics allowed the Transcendentalists to argue that synthetic *a priori* reasoning can lead to indisputable truths — of mathematics, of science, and, they argued, of religion.

3. The strengths of science and mathematics

But how is it that mathematics leads to truth? More generally, how do the natural sciences succeed in describing the world? For the sciences, the answer is simple. Somehow, a researcher thinks of a theory. The manner of his or her inspiration is irrelevant; it can come from a reverie in a meadow, a flash of insight in the lab, an investigation of a failed experiment, a methodical analysis, a fevered dream, or a revelation from God. But however the theory arises, it is tested against reality. If it passes the test, scientists gain confidence in it; if it fails, it is revised or discarded.

Mathematics, unlike the physical sciences, is not tested by its relation to reality. Instead, mathematicians rely on the rigorous logical structure of a *proof*, an argument that must withstand the critical eyes of other mathematicians. Indeed, the *art*

20. Horace Bushnell, "Work and play: Delivered as an oration before the Society of Phi Beta Kappa, in the University of Cambridge," in *Work and play* (New York: Charles Scribner's Sons, 1881), URL: http://books.google.com/books?id=QMoYAAAAYAAJ, 35–36.

21. Also quoted in Cohen, see n. 6, 7, but Cohen does not note that the words are Quinet's.

of mathematics is to take an intuition and to figure out how to fit it, clearly and convincingly, into such an unforgiving logical structure. In the best mathematical proofs, the reader is able to see the intuition behind the formal argument. But a mathematical statement, divinely inspired or no, will not be accepted as proven until it is supported with a rigorous argument.

The Transcendentalists, as we have seen, were very keen on the idea of divinely-inspired synthetic *a priori* reasoning. But they were less eloquent on the necessity of the rigorous proof, or the test against reality. One senses that the Transcendentalists might not have bothered to try to find Neptune in the sky; they would have been satisfied with making the calculations, and trusting that they were correct... or perhaps not even *caring* whether or not they were correct. For instance, in *Nature*, Ralph Waldo Emerson writes:

> Thus even in physics, the material is degraded before the spiritual. The astronomer, the geometer, rely on their irrefragable analysis, and disdain the results of observation. The sublime remark of [Leonhard] Euler on his law of arches, "This will be found contrary to all experience, yet is true;" had already transferred nature into the mind, and left matter like an outcast corpse.[22]

We see that Emerson positively dismisses the importance of testing theories against experience, against reality. Again, from *Nature*:

> But the best read naturalist who lends an entire and devout attention to truth, will see that there remains much to learn of his relation to the world, and that it is not to be learned by any addition or subtraction or other comparison of known quantities, but is arrived at by untaught sallies of the spirit, by a continual self-recovery, and by entire humility. He will perceive that there are far more excellent qualities in the student than preciseness and infallibility; that a guess is often more fruitful than an indisputable affirmation, and that a dream

22. Ralph Waldo Emerson, *Nature* (Boston and Cambridge: James Munroe & Company, 1849), URL: https://books.google.com/books?id=GOOhAAAAMAAJ, 53–54.

> may let us deeper into the secret of nature than a hundred concerted experiments.[23]

It is true that the hard part of science is coming up with an inspired theory; but the inspired theory, the good guess, the dream, must in the end be tested by the precise, careful comparison with reality. Unmoored from these tests, science drifts into alchemy.

In the first of these passages, Emerson is echoing Coleridge, who wrote:

> The celebrated Euler having demonstrated certain properties of Arches, adds: "All experience is in contradiction to this; but this is no reason for doubting its truth." The words *sound* paradoxical; but mean no more than this — that the mathematical properties of Figure and Space are not less certainly the properties of Figure and Space because they can never be perfectly realized in wood, stone, or iron.[24]

(Coleridge makes a similar remark elsewhere.[25]) Coleridge's point is that there is an ideal world that reality merely approximates. But the likely source for this quote from Euler — paragraph 272 of Chapter 3 of *Mechanica*[26] — is one where Euler is considering the equation for the motion of a body on a straight line, subject to acceleration given by a power law. Euler tries to figure out what will happen to the object when it reaches a point on the line where the very assumptions that are the basis for the problem no longer make sense; not just *physical* sense, mind you,

23. Emerson, see n. 22, 64–65.

24. Coleridge, *Aids to reflection, in the formation of a manly character, on the several grounds of prudence, morality, and religion*, see n. 4, 285.

25. Samuel Taylor Coleridge, *The friend: A series of essays, in three volumes, to aid in the formation of fixed principles in politics, morals, and religion, with literary amusements interspersed* (London: Rest Fenner, 1818), URL: http://books.google.com/books?id=tXURAAAAYAAJ, 183.

26. Leonhard Euler, *Mechanica sive motus scientia analytice* (St. Petersburg: Typographia Academiae Scientiarum, 1736), URL: http://eulerarchive.maa.org/pages/E015.html. Here Euler writes *"Hoc quidem veritati minus videtur consentaneum*; [. . .] *Quicquid autem fit hic calculo potius, quam nostro iudicio est fidendum.*" ["Indeed, this [calculation] appears to be less in agreement with the truth; [. . .] In any case, the calculation rather than our judgement is to be trusted."] Note that Euler says that his calculations contradict the *truth*, rather than *experience*.

but *mathematical* sense. It was not uncommon for 18th-century mathematicians to have trouble interpreting the equations they were using; some of their ideas had not been properly formalized yet, and instead of worrying too much about it, they would continue manipulating their equations until they made sense. The comment by Euler can be viewed as more of a statement that he knew that something was wrong, but he wasn't quite sure what… and why not see where the equations will lead?[27]

Coleridge and the Transcendentalists were willing to use mathematics, with its reputation for relevance and utility, to support the idea that there are truths to be obtained through synthetic *a priori* reasoning. But they did not acknowledge that the very source of this relevance and utility lay in the interplay of inspired personal intuition and rigorous arguments subject to examination by others. In the next section, I will discuss how an example of a mathematical truth that they (and others) frequently cited was central to a centuries-old conflict between mathematical intuition and rigor — a conflict that was resolved over the course of the 19th century.

27. It is somewhat strange to find Coleridge, the Romantic, quoting Euler, the scientist and mathematician; one does not imagine Coleridge spending his time reading mathematical treatises. But Coleridge certainly did read Voltaire, and Voltaire quotes these very words of Euler in his *Diatribe du docteur Akakia*. Voltaire refers to the short 1739 pamphlet *Remarks on Mr. Euler's treatise of motion* by the English scientist and engineer Benjamin Robins, which is highly critical of Euler's work on mechanics and which includes this quote (p. 12) to disparage Euler. No love was lost between Voltaire and Euler in the court of Frederick the Great, so Voltaire must have been pleased to be able to include this jab at Euler in his satire. This connection between Euler, Robins, and Voltaire is mentioned by Knobloch (Eberhard Knobloch, "Euler and infinite speed").

Note that the problem Euler is considering has nothing to do with arches. It is possible that Coleridge followed Voltaire's citation and read Robins's pamphlet himself; if this was the case, then perhaps Coleridge was misled by his unfamiliarity with mathematics and by Robins mentioning "arches" (more properly, *arcs* of circles) in the course of his discussion of Euler's work.

4. The angles of a triangle

In his essay *Transcendentalism*, Theodore Parker writes:

> [The science of physics] sets out with other maxims, first truths, which are facts of necessity, known to be such without experience. All the first truths of mathematics are of this character, e.g., that the whole is greater than a part. From these, by the deductive method, it comes at other facts,— facts of demonstration; these also are transcendental, that is, transcend the senses, transcend the facts of observation. For example, the three angles of a triangle are equal to two right angles,— that is universally true; it is a fact of demonstration, and is a deduction from a first truth which is self-evident, a fact of necessity.[28]

Likewise, Coleridge writes "'All the angles of every triangle are equal to two right angles'; this is an apodictic or necessary judgment."[29]

Kant goes to the extreme of outlining a proof of the result stated by Coleridge:

> Suppose that the conception of a triangle is given to a philosopher, and that he is required to discover, by the philosophical method, what relation the sum of its angles bears to a right angle. He has nothing before him but the conception of a figure inclosed within three right lines, and, consequently, with the same number of angles. He may analyze the conception of a right line, of an angle, or of the number three as long as he pleases, but he will not discover any properties not contained in these conceptions. But, if this question is proposed to a geometrician, he at once begins by constructing a triangle. He knows that two right angles are equal to the sum of all the contiguous angles which proceed from one point in a straight line; and he goes on to produce one side of his triangle, thus forming two adjacent angles which are together equal to two right angles. He then divides the exterior of these angles, by drawing a line parallel with the opposite side of the triangle, and immediately perceives that he has thus got an exterior adjacent angle which is equal to the interior. Proceeding in this way, through a chain of inferences, and always on the ground of intuition, he arrives at a clear and universally valid solution of the question.[30]

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28. Theodore Parker, "Transcendentalism," in *The world of matter and the spirit of man: Latest discourses of religion* (Boston: American Unitarian Association, 1907), URL: http://catalog.hathitrust.org/Record/008728028, 24.

29. Coleridge, *Logic*, see n. 7, 260–261.

30. Kant, see n. 9, 524–525.

This geometric fact, proven by Euclid in the manner outlined by Kant — that the sum of the measures of the angles of a triangle is equal to twice the measure of a right angle — plays a role in an important story in the history of mathematics: the discovery of non-Euclidean geometry.

Recall that Euclid's *Elements*[31] begins with 23 definitions ("A point is that of which there is no part," and so on), five axioms ("Things equal to the same are also equal to each other," and so on), and five postulates:

(1) Let drawing a straight line from every point to every point be conceded.

(2) And producing a limited straight line continuously in a straight line.

(3) And drawing a circle for every center and distance.

(4) And all right angles to be equal to each other.

(5) And if a straight line falling upon two straight lines makes the angles within and on the same side less than two right angles, a meeting of the two straight lines, when produced indefinitely, on the side where the angles less than two right angles are.

The fifth postulate stands out as being much more complicated than the others, and much less elegant. Euclid avoids using it as long as possible; he proves 28 propositions before finally getting to one that requires it. Since the time of Euclid, many philosophers and mathematicians, including some of the highest caliber, have tried to derive the fifth postulate from the other assumptions that Euclid makes, with the goal of making geometry more elegant. Boris Rosenfeld[32] provides an extensive annotated history of these attempts.

31. William Barrett Frankland, *The first book of Euclid's* Elements *with a commentary based principally on that of Proclus Diadochus* (Cambridge: Cambridge University Press, 1905), URL: http://books.google.com/books?id=StjuAAAAMAAJ.

32. B. A. Rosenfeld, *A history of non-Euclidean geometry: Evolution of the concept of a geometric space*, vol. 12, Studies in the history of mathematics and physical sciences (New York: Springer-Verlag, 1988), Chapter 2.

The French mathematician Adrien Marie Legendre (1752–1833) was among the many who had an incorrect ‘proof’ of the fifth postulate. He showed (correctly) that Euclid’s fifth postulate is equivalent to the statement that the sum of the angles of every triangle is equal to two right angles; in other words, if you make all of Euclid’s assumptions *except* for his fifth postulate, you can *deduce* the fifth postulate from the statement about the angles of a triangle — and vice versa. Legendre then makes an (incorrect) proof of the statement about triangles. (Rosenfeld[33] discusses Legendre’s failed proofs.)

But the important fact for us is what Legendre *did* successfully prove: The statement about triangles that Kant, Coleridge, and Parker gave as an example of an apodictic fact, a fact of necessity, is equivalent to Euclid’s fifth postulate.

(We should note that there are other statements that are also equivalent to Euclid’s fifth postulate, some of them more concise and more intuitively appealing. One of the more familiar of these equivalent statements is known as “Playfair’s axiom”: Through a point not on a given line, there is at most one line parallel to the given line.)

Now, the *reason* why mathematicians for two millennia had thought they could prove the fifth postulate from Euclid’s other assumptions is that it *seems so clear*, especially when it is formulated as Playfair’s axiom. So if the intuition of mathematicians had told them, for 2000 years, that Euclid’s fifth postulate was clearly true, why could none of them create a proof that was not found to be erroneous?

Because, much to everyone’s surprise, it was shown in the 19th century that it is *logically impossible* to create such a proof. There are models of geometries that satisfy all of Euclid’s axioms and postulates, *except* for the fifth postulate. The earliest of these geometries was discovered by the Russian mathematician Nikolai Ivanovich Lobachevsky, and described by him in an 1829 paper; independently, the Hungarian János Bolyai made the same discovery, and published his work in 1832.

33. Rosenfeld, see n. 32, 103–107.

(Rosenfeld[34] gives a technical account of the history of this work. There are many popular accounts as well; for example, Douglas Hofstadter[35] explores some of the philosophical issues.) It took a number of years for mathematicians to accept these groundbreaking discoveries, and it was not until 1868 that Eugenio Beltrami gave a model for the geometries of Lobachevsky and Bolyai. These geometries do not match our intuition about physical space, which is why it took so long for their existence to be known. In fact, Girolamo Saccheri came close to discovering them in 1733; he attempted to prove the fifth postulate by trying to derive a contradiction from the assumption that the postulate did *not* hold. Saccheri correctly deduced many properties of non-Euclidean space, never finding a technical contradiction (because none exist), but finally deriving a statement (which indeed holds true in non-Euclidean geometry) that he thought was "repugnant to the nature of the straight line."[36,37]

The discovery of non-Euclidean geometry is one of the great triumphs of mathematics. But an equally great triumph is that mathematical standards of proof *prevented* people from accepting Euclid's fifth postulate as a proven fact. The strongly held intuition of many people throughout the centuries was incorrect; and their incorrect intuition was held at bay by the fact that it could not be made to conform to the logical structure of a proof that would convince other mathematicians.

So the example of an apodictic fact given by Kant, Coleridge, and Parker is the central player in a drama in the history of mathematics, a drama that shows that things that are intuitively clear are not always true.

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34. Ibid., Chapter 6.

35. Douglas R. Hofstadter, *Gödel, Escher, Bach: An eternal golden braid* (New York: Vintage Books, 1980), Chapter 4.

36. Rosenfeld, see n. 32, 98.

37. M. C. Escher based some of his woodcuts on non-Euclidean geometry — see Figure 1. The arcs of circles visible in *Circle Limit III* are "straight lines" in Henri Poincaré's model of hyperbolic geometry. The reader is invited to meditate on whether they exhibit repugnant properties.

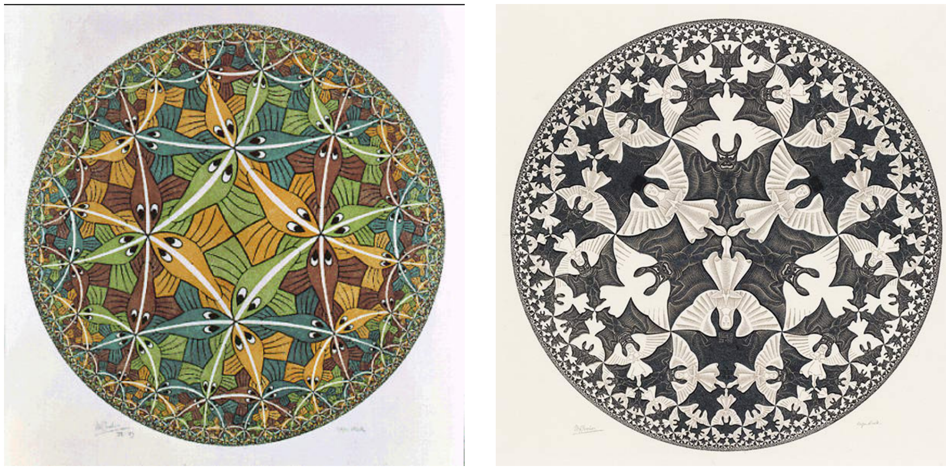

FIGURE 1. *Circle Limit III* (1959) and *Circle Limit IV* (1960), by M. C. Escher. Copyright The M. C. Escher Company B. V., reproduced under educational fair use.

5. The free and responsible search

Where does this leave Transcendentalism? Coleridge's Reason can produce valuable insight, but it can also be mistaken. An answer might be found in one of the seven principles affirmed by Unitarian Universalist congregations: a free and responsible search for truth and meaning.

Descended from the liberal Christianity of the early 19th-century Unitarian churches in New England, the Transcendentalists took freedom of thought and conscience to a new extreme. Emersonian self-reliance in thought, Thoreauvian contemplation and meditative isolation — Transcendentalism invites a radical individualism. This free search for truth and meaning came as a welcome corrective to the more doctrinaire beliefs that liberal Christianity was itself in opposition to. But in the enthusiasm of the new freedom, the Transcendentalists paid less attention to the importance and value of the communal; they emphasized freedom, at the expense of responsibility.

For Unitarian Universalists, the challenge is to balance the radical freedom that is our inheritance from the Transcendentalists with the responsibilities that come with community. Community keeps us true, as we have seen in the example of mathematics; it reminds us that our own truths may not hold universally, and it gives us new perspectives that can change our intuitions. In community, we create worship with people who understand the world differently from us, who challenge the beliefs that seem so intuitively clear to us — people whose intuitive geometry may be different from our own, yet just as correct — people who, despite our differences, will stand with us under the night sky, so that together we may contemplate the motion of the planets.

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